NAG C Library Function Document

nag_zhpcon (f07puc)

1 Purpose

nag_zhpcon (f07puc) estimates the condition number of a complex Hermitian indefinite matrix A, where A has been factorized by nag_zhptrf (f07prc), using packed storage.

2 Specification

```
void nag_zhpcon (Nag_OrderType order, Nag_UploType uplo, Integer n,
     const Complex ap[], const Integer ipiv[], double anorm, double *rcond,
     NagError *fail)
```

3 Description

nag_zhpcon (f07puc) estimates the condition number (in the 1-norm) of a complex Hermitian indefinite matrix A:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a call to nag_zhp_norm (f16udc) to compute $\|A\|_1$ and a call to nag_zhptrf (f07prc) to compute the Bunch–Kaufman factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates how A has been factorized as follows:

if **uplo** = **Nag_Upper**, $A = PUDU^H P^T$, where U is upper triangular;

if **uplo** = **Nag_Lower**, $A = PLDL^H P^T$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **ap**[dim] – const Complex *Input*

Note: the dimension, dim, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

On entry: details of the factorization of A stored in packed form, as returned by nag_zhptrf (f07prc).

5: **ipiv**[dim] – const Integer *Input*

Note: the dimension, dim, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

On entry: details of the interchanges and the block structure of D, as returned by nag_zhptrf (f07prc).

6: **anorm** – double *Input*

On entry: the 1-norm of the **original** matrix A, which may be computed by calling nag_zhp_norm (f16udc). **anorm** must be computed either **before** calling nag_zhptrf (f07prc) or else from a copy of the original matrix A.

Constraint: **anorm** ≥ 0.0.

7: **rcond** – double * *Output*

On exit: an estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

8: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_REAL

On entry, **anorm** = ⟨*value*⟩.
Constraint: **anorm** ≥ 0.0.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_zhpcon (f07puc) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to nag_zhptrs (f07psc) with 1 right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_dspcon (f07pgc).

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A, where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here A is Hermitian indefinite, stored in packed form, and must first be factorized by nag_zhptrf (f07prc). The true condition number in the 1-norm is 9.10.

9.1 Program Text

```
/* nag_zhpcon (f07puc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
  /* Scalars */
  double   anorm, rcond;
  Integer  ap_len, i, j, n;
  Integer  exit_status=0;
  NagError fail;
  Nag_UploType  uplo_enum;
  Nag_OrderType order;

  /* Arrays */
  Integer *ipiv=0;
  char    uplo[2];
  Complex *ap=0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I,J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I,J) ap[(2*n-J)*(J-1)/2 + I - 1]
  order = Nag_ColMajor;
#else
#define A_LOWER(I,J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I,J) ap[(2*n-I)*(I-1)/2 + J - 1]
  order = Nag_RowMajor;
```

```
#endif

  INIT_FAIL(fail);
  Vprintf("f07puc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%*[^\n] ", &n);
  ap_len = n * (n + 1)/2;

  /* Allocate memory */
  if ( !(ipiv = NAG_ALLOC(n, Integer)) ||
       !(ap = NAG_ALLOC(ap_len, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  /* Read A from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            Vscanf(" ( %lf , %lf )", &A_UPPER(i,j).re, &A_UPPER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &A_LOWER(i,j).re, &A_LOWER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  /* Compute norm of A */
  f16udc(order, Nag_OneNorm, uplo_enum, n, ap, &anorm, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f16udc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Factorize A */
  f07prc(order, uplo_enum, n, ap, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07prc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Estimate condition number */
  f07puc(order, uplo_enum, n, ap, ipiv, anorm, &rcond, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07puc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
```

```
    }
  if (rcond >= X02AJC)
    Vprintf("Estimate of condition number =%10.2e\n", 1.0/rcond);
  else
    Vprintf("A is singular to working precision\n");
 END:
  if (ipiv) NAG_FREE(ipiv);
  if (ap) NAG_FREE(ap);
  return exit_status;
}
```

9.2 Program Data

```
f07puc Example Program Data
  4                                                         :Value of N
  'L'                                                       :Value of UPLO
 (-1.36, 0.00)
 ( 1.58,-0.90) (-8.87, 0.00)
 ( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
 ( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00)  :End of matrix A
```

9.3 Program Results

```
f07puc Example Program Results

Estimate of condition number =  6.68e+00
```
